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A DIGITAL EXTREMA-RESOLVING TECHNIQUE

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COMMUNICATION SCIENCES LABORATORY
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ABSTRACT

It is often of interest to determine the extrema (maxima or minima) of a set of measurements. Such a set may be composed of the simultaneous measurements of components of a variable or it may consist of several measurements of a single variable taken over time. Such a set of measurements is by necessity quantized in time, space or in the units of the variable. The digital extrema-resolving technique interpolates the set of measurements in order to locate the extrema and also to resolve the inaccuracies of the extrema locations due to quantization.

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A DIGITAL EXTREMA-RESOLVING TECHNIQUE

This paper is concerned with locating the extrema (maxima or minima) in the measurement of components of a variable. In particular, it relates to locating the maxima of energy in voice frequency spectra. The general application relates to increasing the resolving power of the measuring apparatus of a variable which is to be resolved into its components. For instance, in (astronomical) spectroscopy it is of interest to know which frequency component carries the maximum (or minimum) energy. The digital extrema-resolving technique also permits rougher quantization in space and/or time when extrema measurements of a variable have to be taken. The variable may be pressure, temperature, or density. In crystallography, for instance, the location of the extrema of temperature in crystal growing processes is important.

The particular application will be explained. The rectified and low-pass filtered outputs of a voice spectrum analyzer are multiplexed and converted from analog to digital form. A previous report describes this matter in detail.[1] An eighteen-channel voice-spectrum analyzer, which samples the voice spectrum fifty times per second, produces as output a digital voice-spectrum pattern quantized in frequency and time as shown in the lower half of Fig. 1. In this figure each column is a digital spectrum sample and the energy in the bands is quantized in eight levels, numbered from 0 to 7 inclusive. For clearness the zero's have not been entered in the figure. Smith gives a more detailed discussion of this matter.[2]

It is well known that human speech is a complex wave that can be resolved into several frequencies which contain the energy. It follows that to synthesize artificial speech only the location of the energy peaks on the frequency scale must be known. In speech, the frequency components that carry the energy change continuously as shown in Fig. 1. This fig-

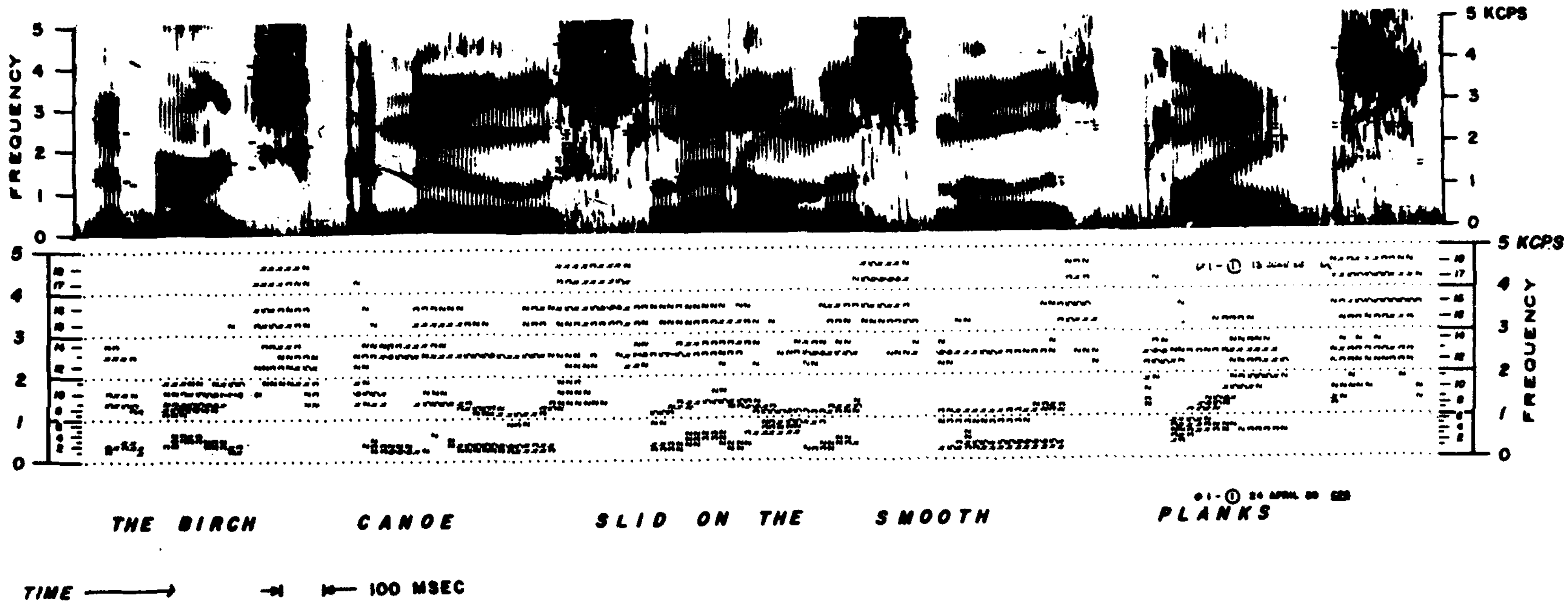

FIG. 1. The results of applying the digital extrema-resolving technique to a digital spectrum pattern.

ure also shows that the energy is spread over several adjacent bands of the spectrum. The inability of the spectrum pattern to limit to one band the position of the energy peaks stems from the nonideal filters in the spectrum analyzer.

The digital extrema-resolving technique interprets the digital spectrum samples in order to locate more precisely the energy peaks. This technique applies a set of rules to each spectrum sample and comes up with a series of ONE's and ZERO's that form a new spectrum sample with a higher frequency resolution. It examines the spectrum sample starting with the number corresponding to the first channel and locates the peak of the energy by the "hill climbing" procedure. It tests the array of numbers in the sample and indicates the peaks by a ONE and the absence of a peak by a ZERO. With this method a spectrum analyzer band is thought to consist of three subbands in one of which an energy peak occurs. The resolution of the analyzer is thereby increased by a factor of three. The channel (band of the analyzer) in which an energy peak falls will be indicated by 010, 100, or 001 depending on whether the peak occurs in the center subband or in one of the side subbands respectively. The channel in which no energy peak occurs will be indicated by 000.

The "hill climbing" method is here used to examine the successive differences of the numbers in the spectrum pattern. A total of 18 differences are obtained for each spectrum sample. A particular difference D_n is defined as:

$$D_n = X_{n-1} - X_n$$

where n refers to the location of the number in the pattern (the channel number of the analyzer) and X represents the amount of energy in the channel. When n is equal to 1 the number X_{n-1} is considered to be equal to zero. When the difference D_n is negative, it indicates that X_n is larger than X_{n-1} and the hill is being climbed. When D_n is positive, then X_n is smaller than X_{n-1} and the hill is being descended. When D_n is equal to zero, than X_n equals X_{n-1} and a flat region is indicated.

To reach the top of the hill it is clear that one or more negative differences must be encountered first. Then, as soon as the first positive difference is encountered we know that the previous difference indicates the channel in which the peak of the energy fell. The previous difference being the last of a series of negative differences indicates the top of the hill. By examining the magnitudes of the last negative difference and the first positive difference, it can be determined in which of the three subbands the energy peak fell. The criteria for locating the proper subbands are:

If $|D_{n-1}| = |D_n|$ then replace X_{n-1} by 010

If $|D_{n-1}| < |D_n|$ then replace X_{n-1} by 100

If $|D_{n-1}| > |D_n|$ then replace X_{n-1} by 001.

For the case in which the top of the hill extends over two or more channels, that is, the outputs of two or more adjacent channels are equal, the rules for locating the energy peak must be extended. If the hill top is flat over two bands, we choose the output code of these two channels to be respectively 001 and 100. A decision for this code cannot be made until the first nonzero difference is positive to ensure that the flat region is a hill top. If the first nonzero difference after the flat region is negative, it indicates that the flat region is a plateau encountered while going up hill. A plateau can also be encountered while going down hill and each of the channels indicating such a plateau must of course be denoted by 000.

Should the hill top be flat over more than two bands, four for instance, then the new digital representation, after applying the described rules, becomes 001 101 101 100. In order to limit the number of all peak energy locations in a band to four, the hill top that is flat over more than two bands will be represented by 001 010 010 100. This ensures coding of all possible peak energy locations in a band by a 2-bit binary number, thereby reducing the transmission rate (bandwidth) when the extrema measurements have to be transmitted.

Figure 2 shows the application of these rules to an arbitrary digital spectrum sample. The first row indicates the number of the spectrum analyzer band. The second row gives the arbitrary spectrum sample. The third row lists the successive differences of the numbers in the second row. The fourth row represents the new spectrum sample with the increased frequency resolution. The upper half of Fig. 1 shows how the digital voice-spectrum pattern of the lower half of Fig. 1 transforms when the digital extrema-resolving technique has been applied. For the sake of clearness the zero's have not been entered in the figure. The upper half of Fig. 1 shows the superposition of the results (applying the digital extrema-resolving technique to the digital spectrum pattern) and the original voice-frequency spectrum. It is shown clearly how the digital extrema-resolving technique tracks the frequencies that carry the peak energy.

In the upper half of Fig. 1 several flat tops are discernible. By choosing either one side or the other of the flat top of the energy peak, tracking can be smoothed. This can be accomplished by examining three successive spectrum samples simultaneously before definitely assigning the new code to that sample that occurs first in time. In other words, first examine the way the peak energy changes before resolving any flat tops. The digital extrema-resolving technique can be applied to a set of measurements by manual computation, by a computer program or it can be instrumented.

CONCLUSIONS

In the measurements of a set of components of a variable the extrema (maxima or minima) can be obtained by interpolating the measurements. This interpolation results in measuring of fewer components because of its inherent quality of increasing the resolution of the measuring apparatus. In speech analyzing apparatus the number of bands can be reduced by a factor equal to the increase-in-resolution factor of the digital extrema resolving technique, without losing the ability to locate the energy peaks in the voice frequency spectrum if the number of bands had not been reduced and the digital extrema-resolving technique had not been used. In the transmission

Number of Spectrum Analyzer Band (n)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Digital Spectrum Sample (X_n)	0	2	6	5	1	0	4	7	7	3	0	6	6	6	1	0	0	2
Successive Differences $D_n - X_{n-1} - X_n$	0	-2	-4	+1	+4	+1	-4	-3	0	+4	+3	-6	0	0	+5	+1	0	-2
New Spectrum Sample, indicating maxima and having higher frequency resolution	000	000	001	000	000	000	000	001	100	000	000	001	010	100	000	000	000	010

FIG. 2. Applying the digital extrema-resolving technique to an arbitrary spectrum sample.

of voice frequency spectrum data obtained from speech analyzing apparatus, or in the transmission of extrema measurements of a set of components of any variable, whereby, respectively, the number of bands or the number of measured components has been reduced, with the application of the digital extrema-resolving technique a significant reduction in the transmission rate and consequently in the bandwidth can be realized.

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